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A CONVECTIVE HEAT TRANSFER COEFFICIENT IN A HIGHLY CIRCULATING REHEATING FURNACE

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Abstract—The measurements of the aerodynamic properties and the convective heat transfer coefficients in the model of a highly circulating reheating furnace are reported. Two methods of measurement of the convective heat transfer coefficient are applied to verify the Chilton–Colburn analogy for the presented geometry. The measurements are compared with the results of the calculation procedure incorporating a two equation turbulence model. The calculation results of the local heat transfer coefficients are presented. The comparison indicates that the calculation procedure qualitatively represents the measurements but that quantitative differences exist.

NOMENCLATURE

a_1, a_2, a_3	coefficients in equation (3)
C	circulation ratio
c_p	specific heat at constant pressure [J kg^{-1} K^{-1}]
D	diameter of furnace [m]
d	diameter of billet [m]
H	height of furnace [m]
h	stagnation enthalpy [J kg^{-1}]
k	kinetic energy of turbulence [m^2 s^{-2}]
m	gas stream [kg s^{-1}]
Pr	Prandtl number
r	radius [m]
Re	Reynolds number
S	source term in equation (3)
s	width of inlet nozzle [m]
Sc	Schmidt number
Sh	Sherwood number
T	temperature [K]
u	radial velocity [m s^{-1}]
W	turbulence parameter [s^{-2}]
w	tangential velocity [m s^{-1}]
y	distance from surface to the nearest grid line [m].

Greek symbols

α	convective heat transfer coefficient [W m^{-2} K^{-1}]
β	convective mass transfer coefficient [kg m^{-2} s^{-1}]
γ	angle [rad]
δ	mass diffusion coefficient [kg m^{-1} s^{-1}]
ε	rate of turbulent energy dissipation [m^2 s^{-3}]
λ	conductive heat transfer coefficient of gas [W m^{-1} K^{-1}]
μ	viscosity coefficient [kg m^{-1} s^{-1}]
ρ	gas density [kg m^{-3}]
σ_T	turbulent Prandtl number
φ	general dependent variable in equation (3)
ψ	stream function [kg m^{-2} s^{-1}]
ω	vorticity [s^{-1}].

Subscripts

e	effective
ex	exit
y	at distance y
w	surface.

1. INTRODUCTION

CONVECTIVE heat transfer to the walls and to the billet plays a considerable role in a highly circulating reheating furnace. Francis and Oeppen [1] have shown that the share of the convective heat transfer in the total heat transfer can reach 90%. Tomeczek and Komornicki [2] have obtained by calculation, that the maximum share of the convection in the heat transferred directly to the billet surface equals 31% and that in the total heat transferred in the furnace equals 92% during the starting-up period. In consequence knowledge of the convective heat transfer coefficients and of the aerodynamics of the furnace is necessary.

Usually such information can be obtained by the model investigation. Davies *et al.* [3] and Bark *et al.* [4] have obtained equations allowing for the calculation of the heat transfer coefficients, but the form in which they have presented the results of experiments does not allow for a unique interpretation. They assumed that the characteristic velocity in the Reynolds number is equal to the velocity on the limit of a boundary layer. Our measurements of the velocity distribution in the model have shown that the value of such characteristic velocity changes along the surface. Moreover, there are considerable differences between the results of refs. [3, 4] in the common range of the Reynolds number. The aim of our research was: (a) to elaborate on the equations for convective heat transfer coefficients which allow for a unique determination of the heat transfer coefficient to the wall and to the billet surfaces; (b) to elaborate on a mathematical model to calculate the velocity profiles and the local heat transfer coefficient in the geometry of a highly circulating reheating furnace; (c) verification of the mathematical model on the basis of the measured velocity for the isothermal flows and of the mean heat transfer coefficient.

In the modelling a known calculating procedure [5], based on the elliptic form of the conservation equations, is assumed. The lack of precise literature data for the analysed geometry required the measurement of the velocity distribution. Different streams of mass in cross-sections of various parts of the analysed geometry are the reason for serious difficulties in the iterative procedure of the mathematical model. These difficulties were partly overcome by using the experimental values of the circulation ratio. We have decided then to present in this paper these experimental values in order to allow the verification of the modelling in future work on this problem.

The investigations of the heat transfer coefficient have begun firstly by the naphthalene method for its simplicity. In the literature for the analysed geometry, the Chilton–Colburn analogy, used for the calculation of the heat transfer coefficient, has not been verified. What is more, there are known cases in which this analogy is not fulfilled [6]. It has been decided then to check some points of this analogy. The lack of agreement has persuaded us to complete full experiments using both the naphthalene method and the direct method of heat transfer coefficient measurement.

2. EXPERIMENTAL INVESTIGATION

The flow patterns and the convection have been studied in the model of the furnace proposed by Davies *et al.* [3]. Figure 1 shows a line diagram of the studied model. The range of parameters and the geometrical dimensions are summarized in Table 1. The parameter which determines the aerodynamics of the furnace is the fluid flow circulating within the furnace. The ratio of the circulating fluid stream to the fluid stream supplied by one inlet nozzle defines the circulation ratio C. Determination of this ratio requires a knowledge of the velocity distribution in the model. The velocity distributions were measured by a constant temperature anemometer for different ratios of d/D, different widths of the inlet nozzle and for several fluid streams at the inlet to the model. No regular correlation between the flow stream and the circulation ratio has been observed. We have noticed however the correlations between the diameter ratio d/D, the width of the inlet nozzle and the circulation ratio. The experimental results presented in

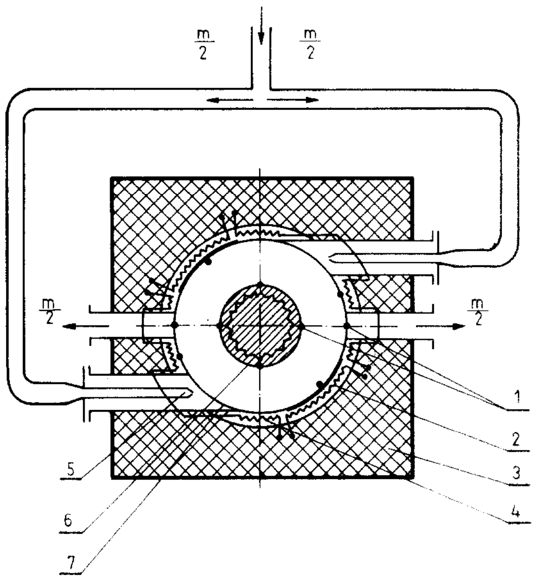

FIG. 1. Diagram of the furnace model: (1) thermocouple; (2) electrical heater; (3) thermal isolation; (4) additional electrical heater; (5) inlet nozzle; (6) billet surface; (7) wall surface of the furnace.

Fig. 2 were obtained for the assumption that the fluid stream from the inlet nozzle does not influence the circulation ratio.

The heat transfer investigations have been done firstly by the naphthalene method because of its simplicity. The heat transfer coefficient determination requires the use of the heat and mass transfer analogy, the most popular of which is the Chilton–Colburn analogy. Neal [6] has shown that the Chilton–Colburn analogy is not fulfilled, even for the flow in an initial part of a tube, and therefore we decided to measure the heat transfer coefficient by the direct method to test this analogy. The first measurements of the heat transfer on the billet surface showed visible deviations from the Chilton–Colburn analogy. Thus we decided to perform systematical investigations of the heat transfer coefficient by both the direct and the naphthalene methods.

Because the geometry of the model is a non-typical one it has been decided, on the basis of the statistical analysis, that the characteristic values of the temperature or of the naphthalene concentration for the determination of the heat transfer coefficient are for both the values at the exit of the model. The statistical

Table 1. The conditions of experiments for: $H = 0.3$ m, $D = 170$ mm, $s = 1.7$ and 5.0 mm

Case	$m \times 10^3$ (kg s^{-1})	d (mm)	T_{ex} (K)	T_w (K)	Remarks
Model surface	10–46	90, 130	293–299	299–315	Heat transfer
		60, 110			Mass transfer
Billet surface		60, 90			Heat transfer
		130			Mass transfer

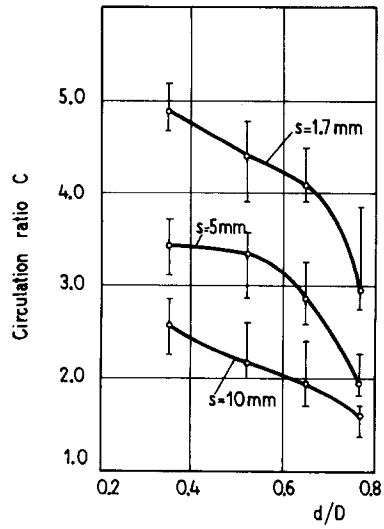

FIG. 2. Experimental values of the circulation ratio.

elaboration of the measured results allowed us to obtain the following correlations [7]

$$Nu = c\ Re^a\ G^b\ Pr^{0.33}, \tag{1}$$

and

$$Sh = c\ Re^a\ G^b\ Sc^{0.33}, \tag{2}$$

where a, b, c, and G are given in Table 2.

The power coefficient 0.33 in these correlations is assumed. Comparison of the obtained correlations has proved that on the wall surface the Chilton–Colburn analogy is satisfactory but on the billet surface there is considerable deviation from the Chilton–Colburn analogy, depending on the Reynolds number. This dependence is similar to that observed by Neal [6] for another geometry and has a qualitative form approximate to the analogy by von Karman and by Spalding–Jayatillaka [8].

3. CALCULATION PROCEDURE OF THE FLOW AND HEAT TRANSFER IN THE FURNACE MODEL

The mathematical model presented below is restricted to the description of the phenomena occurring in the furnace model, therefore it is simplified to the steady, two-dimensional model without radiation heat transfer. The model is based on the stream function, ψ, and on the vorticity, ω, as flow parameters [5]. Because in the flow field a recirculation region appears, the equations describing the flow parameters are therefore elliptic in form. The furnace geometry requires the cylindrical coordinates r and γ. The canonical form of the conservation equation has the form [5]

$$a_1\left(\frac{1}{r}\frac{\partial}{\partial r}\left(\varphi\frac{\partial\varphi}{\partial\gamma}\right)-\frac{1}{r}\frac{\partial}{\partial\gamma}\left(\varphi\frac{\partial\varphi}{\partial r}\right)\right)-\frac{1}{r}\frac{\partial}{\partial r}\left(ra_2\frac{\partial a_3\varphi}{\partial r}\right)-\frac{\partial}{r\partial\gamma}\left(a_2\frac{\partial a_3\varphi}{r\partial\gamma}\right)=S. \tag{3}$$

The corresponding values of a_1, a_2, a_3, and S in equation (3) for the stream functions, vorticity and stagnation enthalpy were taken from ref. [5] and for turbulence parameters from refs. [9, 10]. The differential equations represented by equation (3) were expressed in the finite-difference form of ref. [5] and solved by algorithm of that book. Solution of the conservation equations of stream function and vorticity allowed us to determine the velocity profiles, but because the flow in the model is turbulent it was necessary to complete the mathematical model by a model of turbulence. Two models of turbulence, k–W [9] and k–ε [10], were applied but in the end we based our model on the k–W model which has been easier in this application. Gosman *et al.* [5] has defined the stagnation enthalpy as $h = c_pT+(u^2+w^2)/2+k$, but because the kinetic energy of flow and of turbulence is small in comparison with the thermal enthalpy it was assumed, that the stagnation enthalpy is equal to the thermal enthalpy.

The stream function on a surface is constant. The values of the stream function on the surfaces of the walls of the model and of the billet depend on the gas stream circulating in the model which is unknown at the start of the mathematical modelling. For this reason it has been necessary to supply to the model the experimental values of the circulation ratio, that means a unique determination of the stream function on both of the

Table 2. The values of the constants in equations (1) and (2)

Case	Equation	Re	G	a	b	c
Surface of the wall of the model	$Nu = \frac{\alpha D}{\lambda}$	$\frac{w_{ch}\rho D}{\mu}$	$\frac{D-d}{s}$	0.65	0.594	0.184
	$Sh = \frac{\beta D}{\delta}$			0.624	0.595	0.208
Surface of the cylindrical billet	$Nu = \frac{\alpha d}{\lambda}$	$\frac{w_{ch}\rho d}{\mu}$	$\frac{d}{s}$	0.627	0.221	0.909
	$Sh = \frac{\beta d}{\delta}$			0.759	0.263	0.0302

Note: $w_{ch} = 2m/(D-d)H\rho$.

surfaces. A calculation of the vorticity on the surface, where the stream function is determined, requires the assumption of a distribution of the vorticity within the boundary layer. When a linear distribution of the vorticity is assumed then the correlation between the vorticity and the stream function on the surface has a form

$$\omega_w = -\frac{3(\psi_y - \psi_w)}{r y^2 \rho} - \frac{\omega_y}{2}. \tag{4}$$

as indicated in ref. [5].

The boundary conditions for the stream function were determined according to the notation on Fig. 3 as: constant values on the surfaces AB, EF, CD and GH dependent on the circulation ratio and the gas stream supplied to the model, known functions of r and γ at the inlet nozzle BC, equal to one another on the symmetry lines AG and HF. The boundary condition for the ψ function in the exit from the model DE requires some comments. In order to weaken the influence of the disorder about the exit that results from a sudden change of a velocity direction, it was decided to remove the DE boundary from the corners. The velocity profile on this boundary is assumed then as in a fully developed turbulent flow in a slot. During the calculations the influence of the DE boundary depth on the velocity profiles was investigated and it has been proved that for a sufficient depth its influence is negligible.

The boundary conditions for the vorticity on the surfaces AB, EF, CD, and GH were calculated from equation (4) and on the other boundaries they depend on the distribution of the stream function. The boundary conditions for the turbulent parameters k and W require that the values of these parameters within the boundary layer are determined. The correlations proposed by Spalding [9] were taken for the calculations.

At the inlet nozzle the turbulent parameter k has been taken and the W parameter obtained by recalculation from ref. [11]. At the exit of the model it was assumed that the gradients of k and W were equal to zero. As a result of the assumption that the stagnation enthalpy is limited to the thermal enthalpy, the boundary conditions for the conservation equation of the stagnation enthalpy are equivalent to the boundary conditions of the temperature. Therefore, for the calculations of the heat transfer coefficient on the billet surface a constant temperature was assumed on this surface and an adiabatic condition on the wall surface. At the inlet a constant gas temperature and at the exit a zero gradient of the temperature were assumed.

4. COMPARISON OF CALCULATIONS AND EXPERIMENTS

The present calculations were undertaken with the aim of the verification of the calculation procedure for the present furnace geometry. The calculations were performed with a grid composed of 19 × 32 nodes for several cases, but in this paper the results of a case for the following parameters: $D = 170$ mm, $d = 90$ mm, $s = 5$ mm, $m = 0.01$ kg s^{-1}, are presented. The character of the flow in the model is presented in Fig. 4, where the distribution of the stream function is given. It is clear from this figure, that a recirculation region near the billet surface opposite of the exit does exist.

There are different values of the turbulent constants which are proposed in the literature, therefore calculations were done for the arrangements of the constants recommended by Spalding [9] and Pai *et al.* [12]. The results of these calculations for different arrangements have shown that the constants proposed by Spalding better approach the character of the flow in the model. The calculated values of the tangential velocity are compared with the measurements on Fig. 5. The agreement between the calculated and the measured values may be regarded as satisfactory.

The temperature and the effective viscosity distributions which were obtained by calculations permit us to determine the local heat transfer coefficient. The values of the local heat transfer coefficient were calculated based on the assumption that the heat transferred by convection is equal to the

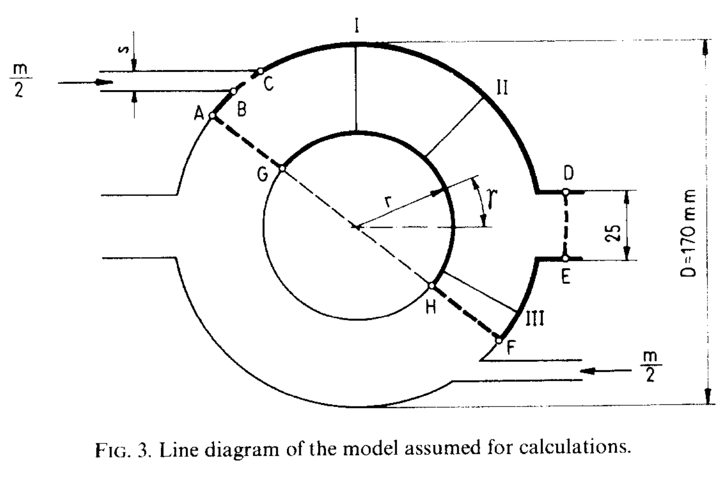

FIG. 3. Line diagram of the model assumed for calculations.

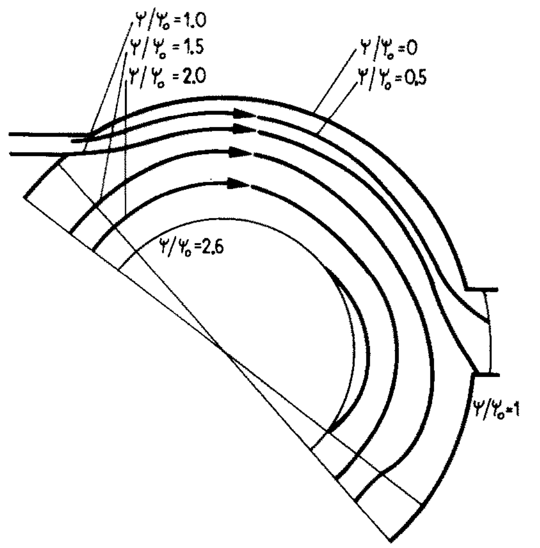

FIG. 4. The calculated stream function distribution for the isothermal flow and for the $d = 90$ mm, $D = 170$ mm, $s = 5$ mm, $m = 0.01$ kg s^{-1}, k–W model of turbulence.

heat transferred by conduction in the boundary layer

$$\alpha(T_w - T_{ex}) = \lambda_e \left|\frac{\partial T}{\partial r}\right|_w. \tag{5}$$

The effective heat conduction coefficient λ_e in a boundary layer was determined by the equation

$$\lambda_e = \left(\frac{\mu}{Pr} + \frac{\mu_e - \mu}{\sigma_T}\right)c_p, \tag{6}$$

where the effective viscosity is determined as the arithmetic mean of the viscosity values on a surface and on the grid line nearest to the surface. The temperature profiles obtained by the calculations are linear near to the surface thus it was simple to determine the gradient of the temperature on a surface. Therefore, the local heat transfer coefficient is determined from the equation

$$\alpha = \left(\frac{\mu}{Pr} + \frac{\mu_e - \mu}{\sigma_T}\right)\frac{c_p}{T_w - T_{ex}}\,\frac{T_w - T_y}{y}. \tag{7}$$

The grid arrangement with smaller grid line distances close to the surfaces was applied. The grid lines at the surfaces were chosen from the demand of accurate calculation of the heat transfer coefficient. For this reason the acceptable distance between the lines has been such that a linear temperature profile could be drawn between the wall temperature and the temperatures of the two nearest grid lines to wall. The velocity profile in the bulk of the flow has been only slightly influenced by the grid arrangement close to the surface.

The value of the turbulent Prandtl number σ_T is of considerable influence on the temperature profile and therefore on the local heat transfer coefficient. In the presented calculations the value of σ_T proposed by Reynolds [13] is applied. The distribution of the local heat transfer coefficient on the billet and on the wall surfaces, which were obtained for $\sigma_T = 0.9$, are shown in Fig. 6. The assumed value $\sigma_T = 0.9$ did allow the best results to be obtained. The distributions of the local heat transfer coefficients are compared on this figure with the values of the mean heat transfer coefficient which was obtained experimentally.

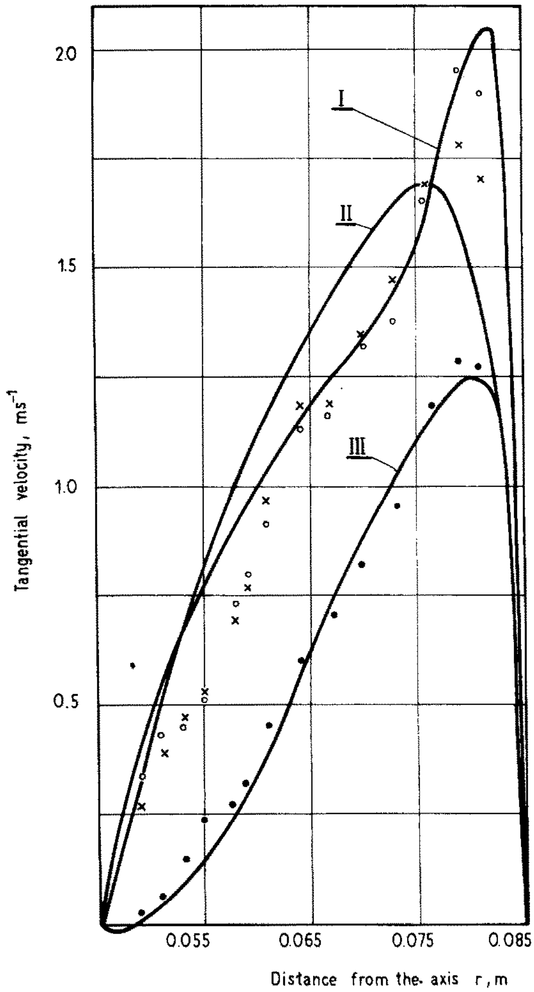

FIG. 5. Comparison of the calculated and measured velocity distribution for the isothermal flow and for the $d = 90$ mm, $D = 170$ mm, $s = 5$ mm, $m = 0.01$ kg s^{-1}, k–W model of turbulence. Curve I, ○ ○; Curve II, × × ; Curve III, ● ●.

For the purpose of comparison of the measured and calculated values of the convective heat transfer coefficient the mean value of the coefficient has been calculated from the local heat transfer coefficient. The calculated mean values, obtained for several chosen cases are presented in Table 3. The analysis of this table shows that there are considerable differences between the measured and the calculated values on the billet surface and much smaller differences on the wall surface. It results from the flow complexity around the billet with the breakage of the boundary layer.

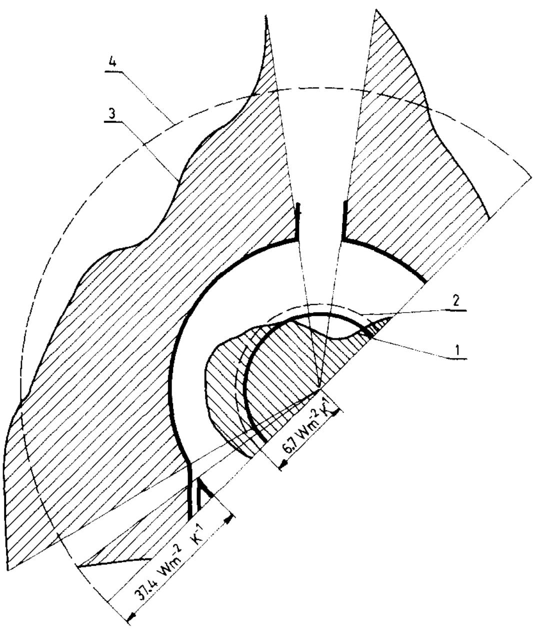

FIG. 6. Comparison of the calculated and measured heat transfer coefficient on the billet and wall surfaces: (1) calculated on the billet; (2) measured on the billet; (3) calculated on the wall; (4) measured on the wall.

5. CONCLUSIONS

The criterion equations which were obtained by means of the model investigations of the heat and mass transfer coefficient cannot be compared with equations presented by Davies *et al.* [3] and Bark *et al.* [4]. It comes from a different definition of the characteristic velocity in the Reynolds number. The equations proposed in this paper give a reasonable estimate of the heat transfer coefficient and can be applied for the design of highly circulating gas reheating furnaces.

The comparison presented in Section 4 shows that the results obtained by the procedure used are in general agreement with the measurements but that deficiencies still remain. This procedure has proved to be a useful instrument to obtain the local heat transfer coefficient. The local heat transfer coefficient measured by Lucas *et al.* [14] on the billet surface for a similar geometry, presents a qualitative likeness to those obtained by calculations which are shown in Fig. 6. Both the calculated and measured distributions are characterized by a strong decrease of the local heat transfer coefficient on the billet surface opposite to the exit. This fact can be explained by the existence of the recirculation of gases in this region. It may be concluded that the present procedure will be helpful at the design calculations of highly circulating reheating furnaces. The completion of this procedure with the radiative heat transfer model would be helpful to get a comprehensive description of the heat transfer phenomena in the furnace.

The Chilton–Colburn analogy is in good agreement with the experiment on the wall surface but the fact that it fails on the billet surface, requires this analogy to be applied with caution.

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Table 3. Calculated and measured mean heat transfer coefficients for $D = 170$ mm, $s = 5$ mm, $T_w = 323$ K

d (mm)	$m \times 10^3$ (kg s^{-1})	Billet surface Calculated	Billet surface Measured	Wall surface Calculated	Wall surface Measured
90	10	7.2	6.66	32.1	37.46
130	10	14.8	13.4	35.3	34.36
130	29	36.3	26.14	69.7	76.82

COEFFICIENT DE CONVECTION THERMIQUE DANS UN FOUR A FORTE VENTILATION

Résumé—On rapporte les mesures des propriétés aérodynamiques et des coefficients de transfert par convection dans le modèle d'un four à forte ventilation. Deux méthodes de mesure des coefficients de convection sont appliquées pour vérifier l'analogie de Chilton–Colburn dans la géométrie considérée. Les mesures sont comparées avec les résultats d'un calcul basé sur un modèle de turbulence à deux équations. On présente les résultats du calcul des coefficients de convection locaux. La comparaison montre que le calcul représente qualitativement les mesures mais qu'il existe une différence quantitative.

KONVEKTIONSWÄRMEDURCHGANGSZAHL IN EINEM WÄRMEOFEN VOM INTENSIVEN ABGASENUMLAUF

Zusammenfassung—In dem vorliegenden Artikel wurden die Messungen der aerodynamischen Eigenschaften und der Wärmedurchflusskoeffizienten im Modell eines Wärmeofens vom intensiven Abgasenumlauf dargestellt. Dabei wurden zwei Methoden für die Messung der Wärmedurchgangszahl angewandt, was die Verifikation der Chilton–Colburn-Analogie für die dargestellte Geometrie erlaubt. Die Ergebnisse dieser Messungen werden mit den aus der Anwendung des Berechnungsverfahrens für die Gasströmung folgenden Ergebnissen verglichen. Dieses Berechnungsverfahren enthalt ein Turbulenzmodell mit zwei Gleichungen. In diesem Artikel wurden auch die Ergebnisse der Berechnungen der lokalen Wärmedurchgangszahl dargestellt. Aus dem Vergleich ergibt sich, dass das Berechnungsverfahren den Charakter der Gasströmung im Wärmeofen widerspiegelt; einige qualitative Unterschiede kommen doch vor.

КОЭФФИЦИЕНТ КОНВЕКТИВНОГО ТЕПЛОПЕРЕНОСА В НАГРЕВАТЕЛЬНОЙ ПЕЧИ С СИЛЬНОЙ ЦИРКУЛЯЦИЕЙ

Аннотация—Проведены измерения аэродинамических характеристик и коэффициентов конвективного теплопереноса на модели нагревательной печи с сильной циркуляцией. Коэффициент конвективного теплопереноса измерялся двумя способами для проверки аналогии Чилтона–Колберна в исследуемой геометрии. Проведено сравнение измеренных значений с результатами расчета по двухпараметрической модели турбулентности. Представлены результаты расчета локальных коэффициентов теплопереноса. Сравнение показывает, что результаты расчета качественно согласуются с измерениями, однако существуют количественные отклонения.